

Structural Equation Modeling

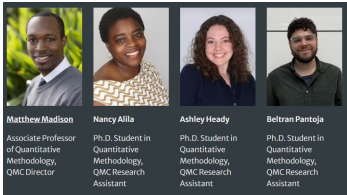
A Quantitative Methods Consulting Center Workshop

Slides and materials available at:
www.matthewmadison.com/workshop



Quantitative Methods Consulting Center (QMC)

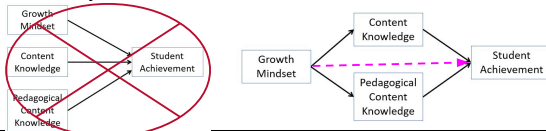
- QMC purpose is to provide quantitative research support and training for members of the MFECOE
- QMC Team



QMC Fall Methods Workshop Series

- "Basics of Measurement Modeling"
 - Introduction to *structural equation modeling* (SEM)
 - Slides and materials available at matthewmadison.com/workshop
- SEM is comprised of two foundational methods:

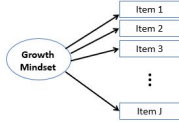
1. Path analysis



QMC Fall Methods Workshop Series

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 - Introduction to *structural equation modeling* (SEM)
 - Slides and materials available at matthewmadison.com/workshop
- SEM is comprised of two foundational methods:

2. Confirmatory Factor Analysis (CFA)

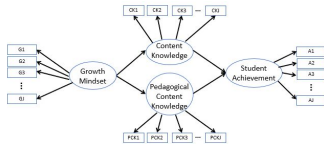


- **Benefits of CFA**
 - Properly model latent variables
 - Assess instrument reliability
 - Assess item quality

QMC Fall Methods Workshop Series

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3. SEM = path analysis + CFA



- **Benefits of SEM**
 - Model complex relationships, test nuanced theories
 - Handle measurement error appropriately
 - Improved power and inferential accuracy
 - SEM is foundational for many other methods and techniques

Structural Equation Modeling

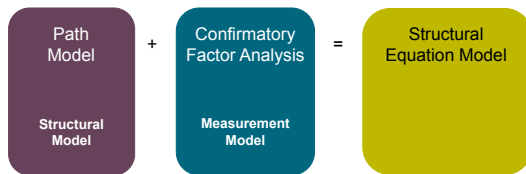
A Quantitative Methods Consulting Center Workshop

Ashley Heady, Quantitative Methodology

Agenda

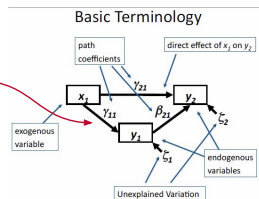
1. The Structural Equation Model
2. Building an SEM
3. Estimating SEM using R
 - a. Evaluating & tuning model fit
4. Interpretation
5. How to Choose a Model
6. Resources & Extras

How it all goes together



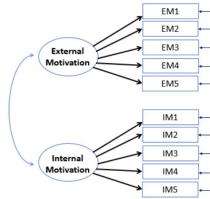
Refresher of Path Models

- Path models are the structural models in SEM
- They specify a directional relationship or a theorized structure between observed variables



Refresher of Confirmatory Factor Analysis

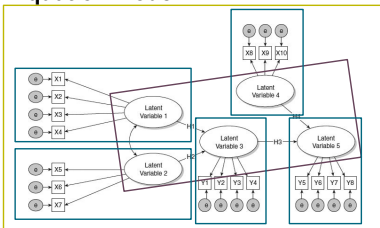
- CFA's are the measurement model part of the SEM
- They specify a correlational relationship between observed and latent variables
- Factors should have at least three indicators (for one factor models), or at least two indicators (for models with more than one factor)



The Structural Equation Model

- Structural Model (path model)
- Measurement Model (CFA)
- Structural Equation Model

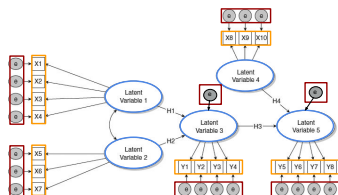
✓ Latent variables now have direct and indirect effects on each other



The Structural Equation Model

- Factors/Latent Variables
- Indicators/Items
- Error
- Measurement Error
- Structural Error

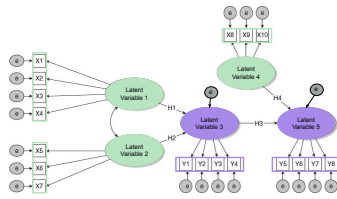
Take a look at the shapes!
□ = observed
○ = latent



The Structural Equation Model

Exogenous variables
(X's, observed & latent)

Endogenous variables
(Y's, observed & latent)



Building an SEM

Building an SEM

- Get to know your data first
 - What do scores in your data set mean?
 - What do the distributions look like? (make some histograms!)
- Extreme differences in variance becomes a problem in SEM
 - Simple fix: rescaling (scale()) in R
- Run a CFA first
 - Check that your items are functioning the same across groups (think gender, grade, or ethnicity groups)
- Identification
 - Identification in SEM has to do with the amount of information we have vs the number of parameters we're estimating

Kang, H., & Ahn, J.-H. (2011). Model Setting and Interpretation of Results in Research Using Structural Equation Modeling: A Checklist with Guiding Questions for Reporting. *Asian Journal of Research*, 1(1), 137-142.
<https://doi.org/10.1016/j.asj.2011.06.001>

Understanding SEM Identification

Identification starts with the t rule: $t \leq \frac{1}{2}(q)(q+1)$

→ basically the number of estimated parameters must be less than or equal to the observed unique entries in the covariance matrix

1

CFA Identification

- Three rule indicator
 - At least three indicators per latent variable
 - Indicators load to only one latent variable
 - No correlated errors

2

Structural Identification

- Recursive rules:
 - Are there direct effects with correlated structural error?
 - Do Y's have reciprocal effects?
- If no for both of the above - model is identified



Understanding SEM Identification

Identification starts with the t rule: $t \leq \frac{1}{2}(q)(q+1)$

→ basically the number of estimated parameters must be less than or equal to the observed unique entries in the covariance matrix

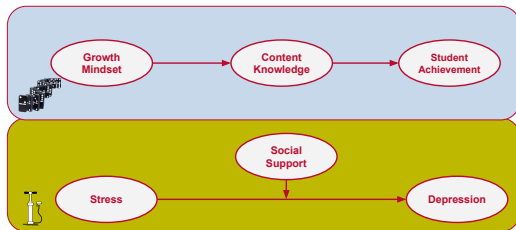
1

What happens if my model isn't identified?

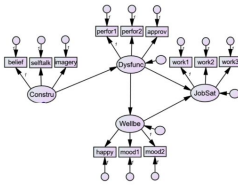
Parameters won't be stable (they'll change every time you run the model) so your output won't be interpretable 😞



Mediation vs. Moderation



Example SEM - Direct Relationships Hypotheses



Constr = Constructive thought strategies

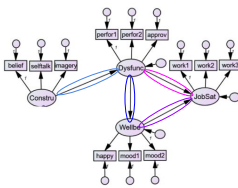
Dysfunc = dysfunctional thought process

Wellbe = Subjective well-being

JobSat = Job satisfaction

Boughton, J.D., Zinkerman, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. *J Bus Psychol* 22, 45-53 (2007). <https://doi.org/10.1007/s10569-007-9046-9>

Example SEM - Direct Relationships Hypotheses



Hypothesis 1
Subjective well-being will be positively related to job satisfaction.

Hypothesis 2
Dysfunctional thought processes will be negatively related to subjective well-being.

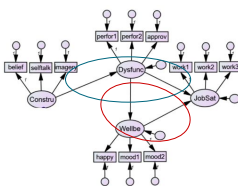
Hypothesis 3
Constructive thought strategies will be negatively related to dysfunctional thought processes....

Hypothesis 4
Dysfunctional thought processes will be negatively related to job satisfaction.

How do we test these hypotheses?
→ Check the significance of the regression coefficients

Boughton, J.D., Zinkerman, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. *J Bus Psychol* 22, 45-53 (2007). <https://doi.org/10.1007/s10569-007-9046-9>

Example SEM - Indirect/Mediating Relationships



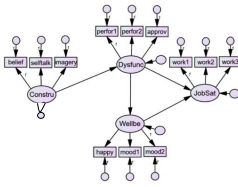
Hypothesis 5
The effects of dysfunctional thought processes on job satisfaction will be partially mediated through subjective well-being.

Hypothesis 6
The effects of constructive thought strategies on job satisfaction will be fully mediated through dysfunctional thought processes and subjective well-being.

How do we test these hypotheses?
→ Compare models with & without mediation

Boughton, J.D., Zinkerman, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. *J Bus Psychol* 22, 45-53 (2007). <https://doi.org/10.1007/s10569-007-9046-9>

Example SEM - Identification



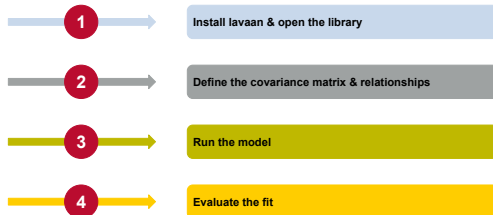
t-Rule
✓ 28 parameters ≤ 78 unique entries in covariance matrix ($(12 \times 13)/2$)

Three-indicator rule
✓ Three indicators per latent variable
✓ Indicators load to only one latent variable
✓ No correlated errors

Structural Identification
✓ No recursive relationships
✓ No correlated structural error

Estimating SEM in R

Estimating Path Models in R



The Code

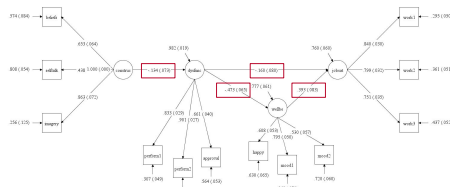
```
install.packages(lavaan); library(lavaan)
```

```
sem_model <- '
construct =~ beliefs + selftalk + imagery
dysfunc =~ perform1 + perform2 + approval
wellbe =~ happy + mood1 + mood2
jobstat =~ work1 + work2 + work3
dysfunc ~ construct
wellbe ~ dysfunc
jobstat ~ dysfunc + wellbe'
```

```
fit <- sem(sem_model, sample.cov=cov, sample.nobs=263)
```

```
summary(fit, fit.measures=TRUE, standardized=TRUE)
resid(fit,type='standardized'); modindices(fit)
```

Example SEM - with Estimates



Boughton, J.D., Zinkernagel, D.L. Constructive Thought Strategies and Job Satisfaction: A Preliminary Examination. *J Bus Psychol* 22, 43-53 (2007). <https://doi.org/10.1007/s10869-007-9046-9>

Model Fit Evaluation

Chi-Squared

Null hypothesis
is essentially
"the model fits"

RMSEA

Two outputs:
> RMSEA estimate
> RMSEA Confidence
Interval

CFI/TLI

CFI & TLI compare your
proposed model to the
baseline model

Looking for:
- $p\text{-value} > 0.05$

Looking for:
- $RMSEA \leq 0.05$

Looking for:
- $CFI \text{ \& } TLI \geq 0.90$

Model Tuning

Residuals

> Shows you the difference between your actual covariance matrix and the fitted model covariance

Looking for:

- Values > 2 are problematic

Modification Indices

> Shows you paths and covariances that could be estimated in the model that would make the model fit better

Looking for:

- Values > 4 → let the model estimate


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Estimating SEM in R 28

Things to remember in SEM

- Draw out your model first! How does it look?
- Complex models are hard to interpret & require lots of data! Keep it simple
- Remember to look at all the fit statistics, they come as a package (but remember chi-squared is very sensitive!)
- Your proposed model does not prove causality, it's just an assumption of how things work


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How to Choose a Model

Choosing a Model

Least Complex

Most Complex

	Multiple Regression	Path Model	CFA	SEM
Goal	Description or Prediction	Observational Structure	Instrument Reliability	Complex Latent Relationships
Sample Size	~30-50+	~60-100+	~90-400+**	~90-400+**
Inferences	Observational	Observational	Latent	Latent

**Wolfe, E. J., Harrington, K. M., Clark, E. L., & Miller, M. W. (2013). Sample Size Requirements for Structural Equation Models: An Evaluation of Power, Bias, and Solution Property. *Educational and psychological measurement*, 76(6), 913-934. <https://doi.org/10.1177/0013164413505237>

A note on CFA & SEM Sample Size

Proper sample size is impacted by:

- Number of factors
- Indicators per factor
- Missingness
- Strength of relationships

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Sample Size*

Indicators	Number of Factors		
	1	2	3
3	190	460	420
6	90	190	190
8	90	170	160

*Assuming no missing data and moderate relationships (loadings = .50)

**Wolfe, E. J., Harrington, K. M., Clark, E. L., & Miller, M. W. (2013). Sample Size Requirements for Structural Equation Models: An Evaluation of Power, Bias, and Solution Property. *Educational and psychological measurement*, 76(6), 913-934. <https://doi.org/10.1177/0013164413505237>

Resources & Extras

Resources for Path Models & SEM

Lavaan Website
lavaan.ugent.be/
tutorials & resources

YouTube
CenterStat,QuantFish,
our QMC YouTube channel

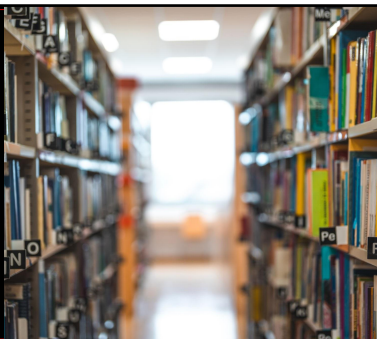
Courses:
- **ERSH 7600:** construction of measurement instruments
- **ERSH 8130:** survey of latent variable methods
- **ERSH 8750:** structural equation modeling
- **ERSH 8740:** exploratory factor analysis

Books:
- **Kline:** Principles and practice of structural equation modeling
- **Bollen:** Structural equations with latent variables
- **Loehlin:** Latent variable models: An introduction to factor, path, and structural equation analysis

More Questions?

Visit us at the Quantitative
Methods Consulting Center!

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Thanks for coming!
